HW Assignment #3: Due Thursday March 6th, 2003

The following problems are due 3/6/03. Please write out complete solutions, with all assumptions and steps written out explicitly, and feel free to email me if you have questions.

1) In the first homework assignment, you learned about the rotation curves associated with a point mass (Keplarian), constant density mass distribution (rigid body rotation), and a $1/r^2$ mass density distribution (flat rotation curve). Now consider a flat (two dimensional) disk galaxy whose surface brightness declines exponentially, i.e., $I = I_0 e^{-r/r_0}$, where $r_0$ is the galaxy’s scale length. Assume that the mass-to-light ratio over the entire galaxy is constant.

   a) Show that the total mass of the galaxy is given by $M_{\text{Total}} = 2\pi \Gamma I_0 r_0^2$, where $\Gamma$ is the mass-to-light ratio.

   b) Determine and plot the rotation curve of the galaxy. Where does the maximum velocity occur?

2) The surface brightness profile of an elliptical galaxy is:

   $$\log \left( \frac{I}{I_e} \right) = -3.33071 \left[ \left( \frac{r}{r_e} \right)^{1/4} - 1 \right]$$

   Show that this reduces to equation 23.2 in C&O.

3) For the galaxies in the Virgo Cluster, the dispersion in radial velocity is 666 km/s. Use the Virial Theorem to estimate the mass of the Virgo cluster, assuming a spherically symmetric constant density mass distribution. Assume a radius of 1.5 Mpc.

4) Assuming all elliptical galaxies have the same mass-to-light ratio, use the virial theorem to show that a relationship should exist between the galaxy’s luminosity and the fourth root of the velocity dispersion.