(2) Cont'd...

We have
\[
\log \left( \frac{\Delta L}{L_e} \right) - 3.3 = \frac{\Delta M_e - \Delta M}{2.5}
\]

where \( \Delta M \) and \( \Delta M_e \) are the absolute magnitudes of the small region of the galaxy associated with \( M \) and \( M_e \), i.e., it is the magnitude per unit surface area of \( M \) and \( M_e \), respectively.

Rewriting above:
\[
\frac{\Delta M_e - \Delta M}{2.5} = \frac{M - M_e}{2.5} = -3.3307 \left[ \left( \frac{r}{R_e} \right)^{1/4} - 1 \right]
\]

which reduces to eq. 23.2 in the book (Caroll & Ostl)

(3) The virial theorem is:

\[ M = 5R^2 \left< r^2 \right> / c_x \]

for a uniform spherical distribution of radius \( R \) and radial vel. dispersion \( \left< r^2 \right> \).

Using \( R = 1.5 \) Mpc,
\[ c_x = 666 \text{ km/s} \]

\[ M = 5 \left( 1.5 \times 10^4 \right) \left( 3.08 \times 10^{18} \right) \left( 666 \times 10^8 \right)^2 \]

\[ \text{MD} \frac{0.67 \times 10^{-8} \times 2 \times 10^{-33}}{c_x} \]

\[ M = 7.7 \times 10^{14} \text{ MD} \]

(4) The virial theorem implies:

\[ r^{-4} \propto \frac{M}{R^2} \]

If the mass-to-light ratio is roughly the same for all elliptical galaxies then

\[ M/L = K_{ML} = \text{constant} \]

Thus

\[ \Sigma_r ^4 \propto K_{ML} \frac{L^2}{R^2} \]

If we assume that the average surface brightness is also constant for all ellipticals, \( \Sigma_r \propto L \)

Therefore

\[ L \propto \Sigma_r ^4 \]