Homework Assignment 5: Due Thursday April 10, 2003

1. In class we studied the behavior of the forbidden-line luminosity coming from a transition corresponding to a 2-level atom, i.e., cases where only the two lowest levels are populated. In this homework problem, assume that the level populations of a certain ion are determined by statistical equilibrium between collisional excitation, collisional de-excitation, and spontaneous emission among the three lowest energy levels. This is a much better approximation than the one we used in class. Denote the line luminosities from levels 2-1 and 3-1 as $L_{21}$ and $L_{31}$, respectively. You are about to investigate how the line luminosity ratio ($L_{21}/L_{31}$) can be used as a diagnostic of the electron density of the ionized gas.

   (a) Show that the line luminosity ratio approaches a constant as the density approaches zero, and approaches a different constant as the density approaches infinity. Derive expressions for these limiting values in terms of the Einstein $A$ coefficient, and the collisional strengths, $\Omega$ for the transitions. To simplify the algebra, you can assume that you can ignore transitions into level 2 from collisional de-excitation and radiative decay from level 3, and that you can also ignore collisional excitations from level 2 into level 3. (10 pts)

   (b) Consider the specific case of Ne$^{+3}$ with level 1 = $^4S_{3/2}$, 2 = $^2D_{5/2}$, 3 = $^4D_{3/2}$. The $A$ coefficients for these transitions are: $A_{32} = 1.4 \times 10^{-6}$ s$^{-1}$, $A_{21} = 5.9 \times 10^{-4}$ s$^{-1}$, $A_{31} = 5.6 \times 10^{-3}$ s$^{-1}$. The collisional strengths for the transitions are $\Omega_{31} = \Omega_{13} = 0.56$ and $\Omega_{21} = \Omega_{12} = 0.84$. Assume that the temperature is 8000 K. What are the critical densities of each of the two transitions? Sketch the approximate behavior of the line luminosity ratio with increasing density, and comment on the range of densities for which these lines are a useful density diagnostic. (10 pts)

2. In a certain Seyfert galaxy that you observe, you find that there are no broad [NeIII] 3967.5 angstrom lines in the spectrum. However, you do detect a broad [FeX] 6374.6 angstrom line. What does this tell you on the likely limits to the density of the broad line region in this galaxy? Explain. Lecture notes will be helpful for this problem. (10 pts)

3. Consider a thin accretion disk orbiting a black hole. Using the ordinary Newtonian formula for the orbital velocity, calculate the radius in the disk (in units of the Schwarzschild radius) where the orbital velocity is 10,000 km s$^{-1}$. If the black hole has a mass of $10^8$ Solar masses, and is accreting at the Eddington limit, calculate the effective temperature of the disk at this radius. Use your result to explain why the broad emission lines seen in the optical spectra of some AGN have characteristic widths of $\sim 10^4$ km s$^{-1}$ (rather than, say, $\sim 10^5$ km s$^{-1}$). (15 pts)

4. This question explores how fast black holes can grow via accretion, under the assumption that the maximum accretion rate is governed by the Eddington limit. Explain
what is meant by the *Eddington limit* on the luminosity. What are the main assumptions that are usually made in deriving the Eddington limit? (*5 pts*)

(i) Consider a black hole accreting mass through a thin disk with a constant efficiency \( \eta = 0.1 \). As the black hole grows, it accretes gas at a rate such that its luminosity is always given by the Eddington limiting value. Under these circumstances, show that the rate of growth of the black hole mass \( M \) is described by,

\[
\frac{dM}{dt} = kM
\]

and calculate the value of the constant \( k \). (*5 pts*)

(ii) Solve this differential equation and show that the mass of the black hole after time \( t \) is given by the expression,

\[
M = M_0 e^{t/\tau}
\]

where \( M_0 \) is the mass at \( t=0 \) and \( \tau \) is a constant, called the *Salpeter time*. Calculate the value of \( \tau \) in years. (*5 pts*)

(iii) A luminous quasar is observed at high redshift, at an epoch when the age of the Universe was only 500 Myr. It is estimated that the black hole mass is \( 10^9 \) Solar masses. Could the quasar have grown from a 10 Solar mass seed in the available time, assuming accretion limited by the Eddington limit? (*5 pts*)