The Production of Polarization

Today we will talk about the production of polarized light. We already introduced the concept of the polarization of light, a transverse EM wave.

To briefly review (see Lecture 3):

A. Linear or Plane Polarization

The ±-direction of $\vec{E}$ (or $\vec{B}$) stays constant in time.

Let

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

Generally,

$$\vec{E}_0 = E_{0x} \hat{i} + E_{0y} \hat{j}$$

$$\text{Re}(\vec{E}) = E_{0x} \cos(kz - \omega t) \hat{i} + E_{0y} \cos(kz - \omega t) \hat{j} = \left( E_{0x} \hat{i} + E_{0y} \hat{j} \right) \cos(kz - \omega t)$$

No time dependence

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**Linear Polarization Schematic:**

**B. Unpolarized**

The direction of $\mathbf{E}$ varies randomly with time.

**C. Circular Polarization**

The $\mathbf{E}$ - (and $\mathbf{B}$ -) field rotate with time. We write the electric field as

$$\mathbf{E} = (\hat{i} + \hat{j})E_0 e^{i(kz-\omega t)} = (\hat{x} + i\hat{y})E_0 e^{i(kz-\omega t)}$$

From Euler’s equation for complex variable

$$e^{i\theta} = \cos \theta + i \sin \theta$$

We will substitute in for the exponential on the right-hand side to get
\[
\vec{E} = (\hat{x} + i\hat{y})\vec{E}_0 e^{i(kz-\omega t)} = (\hat{x} + i\hat{y})\vec{E}_0 (\cos(kz-\omega t) + i \sin(kz-\omega t)) \\
= \vec{E}_0 (\cos(kz-\omega t)\hat{x} + i \sin(kz-\omega t)\hat{x} + i \cos(kz-\omega t)\hat{y} - \sin(kz-\omega t)\hat{y}) \\
= \vec{E}_0 (\cos(kz-\omega t)\hat{x} - \sin(kz-\omega t)\hat{y} + i (\sin(kz-\omega t)\hat{x} + \cos(kz-\omega t)\hat{y}))
\]

Hence,

\[
\text{Re}(\vec{E}) = \vec{E}_0 \cos(kz-\omega t)\hat{x} - \vec{E}_0 \sin(kz-\omega t)\hat{y}
\]

We see the \(x\)- and \(y\)-components are 90° out of phase and the magnitude, \(E_0\), is independent of time.

To see what is happening in this complex situation, let us fix \(z = 0\) and vary time.

Looking into an on-coming wave, \(\vec{E}\) rotates anticlockwise. This is left-handed circular polarization.

**Circular Polarization Schematic:**
**Production of Polarized Light**

How do we produce polarized light? Any interaction of light with matter whose optical properties are asymmetric along directions transverse to the propagation vector \( \Rightarrow \) polarized light.

We will consider the following processes that produce polarized light:

1. Dichroism (or, selective absorption)
2. Reflection
3. Scattering
4. Bifringence (or, double refraction)

**1. Dichroism**

A dichroic polarizes by selectively absorbing light with \( \vec{E} \) field along a unique direction characteristic of the dichroic material.

Simplest such device: **Wire grid polarizer**

![Output light contains only x-polarization.](image)

As shown above, this consists of simply parallel conducting wires where grid spacing \( \ll \lambda \) (can do this for microwaves). Let us suppose we have unpolarized light incident on this grid.
The $\vec{E}$ field can be resolved into 2 orthogonal coordinates in the x and y directions. Let us say that the y-axis is chosen to be along the wires.

Then, $\vec{E}_y$:
- Drives conduction electrons along wires.
- Gives rise to a current. Energy transferred from field to wire (wire heats up)
- Accelerating electrons radiate like dipoles
- Dipoles do not radiate along dipole axis
- They radiate in forward and backward direction
- It turns out that the forward direction is emitted at 180° out of phase from incident radiation
- Results in cancellation of forwarded transmitted component along y-direction
- The backward component is the reflected ray
- $\vec{E}_x$ cannot similarly drive electrons into oscillation since it is perpendicular to the wires in grid

**Net result:** $\Rightarrow$ Little transmission of y-component. x-component transmitted unaltered.
$\Rightarrow$ You get polarized light out with direction perpendicular to wire grid.

**For optical wavelengths**

Similar idea but conduction paths analogous to wires in grid must be much closer.

Most common polarizer: “Polaroid H-sheet” invented by Edwin Land.

This is a clear sheet of polyvinyl alcohol heated and stretched along one direction.

Hydrocarbon molecules align along direction of stretching. Material then dipped in iodine solution $\Rightarrow$ provides “conduction” electrons $\Rightarrow$ molecular chains formed (analogous to wires in grid). Transmission axis is perpendicular to direction of stretching.
If linearly polarized light is incident on the polarizer and the angle between the \( \vec{E} \) field and the transmission axis of the polarizer is \( \theta \), then we have:

\[
\begin{align*}
\vec{E}_{\text{transmitted}} &= \vec{E}_0 \cos \theta \\
I_{\text{transmitted}} &= I_0 \cos^2 \theta
\end{align*}
\]

This is called \textit{Maus’ Law}.

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2. Reflection

This is the principle behind Polaroid sunglasses. It is one of the most common forms of polarized light.

Consider the case where a linearly polarized wave is incident on an interface where \( \vec{E} \) is perpendicular to the plane of incidence as shown below:

The \( \vec{E} \) field drives the bound electrons in the medium, in this case perpendicular to the plane of incidence (perpendicular to the page). \( \Rightarrow \) they radiate. Some of this radiation is radiated along the reflected beam direction = reflected beam.

\( \vec{E}_r, \vec{E}_i \) are also perpendicular to the plane of incidence as shown in the Figure.

Now consider the case where \( \vec{E} \) is parallel to plane of incidence as shown below. \( \vec{E} \) field in transmitted medium is perpendicular to direction of propagation and also parallel to the incident plane. This is the direction that the oscillators are driven (note: direction of beam can be calculated via Snell’s law).

Notice that in this case, the reflected ray direction makes a small angle \( \theta \) wrt. to dipole axis of refracted ray \( \Rightarrow \) little radiation is emitted from a dipole \( \perp \) to dipole axis. If \( \theta = 0^\circ \) \( \Rightarrow \) \( \theta_r + \theta_i = 90^\circ \). In this case, reflected ray would vanish entirely. (Why? Because reflected ray direction is fully parallel to dipole axis \( \Rightarrow \) no radiation).
The particular angle of incidence where this occurs is called “Brewster’s angle” \( \equiv \theta_P \)

\[ \theta_i = \theta_r = \theta_P \quad \text{when} \quad \theta_p + \theta_r = 90^\circ \]

So, using Snell’s Law:

\[ n_i \sin \theta_p = n_r \sin \theta_i \]

But, \( \theta_i = 90^\circ - \theta_p \), so we have:

\[ n_i \sin \theta_p = n_i \cos \theta_p \]

\[ \tan \theta_p = \frac{n_r}{n_i} \] \( \Rightarrow \) Brewster’s Law

When \( n_i = 1 \) (air) and \( n_r = 1.5 \) (glass) \( \Rightarrow \) \( \theta_p \approx 56^\circ \)

So, at this angle of incidence, only the component of \( \vec{E} \) perpendicular to the plane of incidence is reflected. No reflection of component of \( \vec{E} \) parallel to the plane of incidence.

3. Scattering

Scattering of light off air molecules produces linearly polarized light in a plane perpendicular to incident light. Why? Can think of scatterers (molecules in atmosphere) as dipole radiators:
Remember, dipole radiator emit max. radiation perpendicular to dipole axis (no radiation along axis of oscillation). Therefore, 90° away from beam of incidence direction, scattered light will be linearly polarized. Vibrations from particles radiate to observers only when vibration is perpendicular to observer’s line of sight, so light will be polarized along direction of vibration.

4. Bifringence

There are materials displaying asymmetric behavior where the speed of the wave in one direction is different than in the perpendicular direction → two different values of index of refraction, $n_x$ → two refracted beams. These materials are said to be bifringent.

Let’s say you shine light through a plate of bifringent material.

Let’s say $\vec{E}$ is parallel to optic axis → light will travel with some velocity $v$.
Let’s say $\vec{E}$ is perpendicular to optic axis → light will travel with some different velocity $v'$.

Let’s say $\vec{E}$ is at 45° to optic axis. Can break up into equal amplitude $x$ and $y$ components. Since $\vec{E}_x$ and $\vec{E}_y$ travel at different speeds, if they start out in phase, they will not continue to be in phase as they travel through the material.

Result: polarization changes as wave passes through plate.

If thickness of plate is chosen such that there is a 90° phase shift between $\vec{E}_x$ and $\vec{E}_y$ when they emerge → get circularly polarized light out. This is called a quarter wave plate because it produces a $\lambda/4$ phase shift. This is illustrated below:
If the thickness of the plate is \( d \), the thickness of a quarter wave plate would be given by:

\[
n_1d - n_2d = \frac{\lambda}{4}
\]