Laser is a source of light. It differs from other sources of light, such as a light bulb, in a fundamental way: it is a coherent source of light. This property makes laser light more directional, more monochromatic, and hence brighter compared with so-called thermal sources.

Thermal sources of light.

Before we review lasers, let us review thermal emission. When a system is in thermal equilibrium with its surroundings, there is no net energy flow between the system and its surroundings. It can be characterized by a common temp. T. (measure of internal KE of constituent atoms or molecules)

Atoms (or molecules) contain charge particles in motion \( \Rightarrow \) accelerated charges radiate.

For an object that absorbs all incident energy and re-emits it, the intensity of radiation is given by the Planck formula:

\[
\frac{I}{\lambda^5} = \frac{2\pi \hbar c^2}{\lambda^5} \left( \frac{1}{e^{h\nu/kT} - 1} \right) 
\]

(\text{power/area/side})

Such an object is called a black-body and is a whose approximation to Sun (stars), you, me, etc.
This function looks like this:

\[ T = 6000\, \text{K} \quad (\text{peak}) \]

\[ T = 4000\, \text{K} \quad (\text{observable}) \]

\[ T = 3000\, \text{K} \]

Peak of function occurs:

\[ \lambda_{\text{max}} = 2.898 \times 10^2 (\mu\text{m}\cdot\text{K}) \]

\[ \Rightarrow \text{Wien's Law} \]

Total power radiated over all \( \lambda \):

\[ P = \int \sigma A T^4 \]

\[ \sigma = 5.67033 \times 10^{-8} \text{ W/m}^2\text{K}^4 \]

\[ \Rightarrow \text{Stefan-Boltzmann Law} \]

This is the process by which thermal sources radiate.

\( T\) max is not a coherent process.
Interactions of Radiation with Matter:

To understand lasers, one must understand how light interacts with matter. There are 3 fundamental processes:

1) Absorption

2) Spontaneous emission

3) Stimulated emission

Stimulated emission is a coherent process (emitted photons have the same frequency, the same direction, the same phase).

Two-Level Atoms:

Atoms of any material can exist in several energy states. The lowest energy level $E_1$ is called the ground state and represents the relaxed state. The atom can be excited to higher energy states if it is supplied with the required energy $[E = h\nu = E_2 - E_1]$. 

$E = (1 + \alpha) E_0$
In thermal equilibrium, atoms are populated according to the Boltzmann distribution:

\[ N_i = \frac{N_0 e^{-E_i/kT}}{\text{constant}} \]

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This states the higher the excited energy state, the fewer atoms will be in that state.

The relative populations are given by:

\[ \frac{N_i}{N_j} = e^{-E_i/kT} \]

[e.g. at room temp. (T = 300K) \( N_2 = 4.1 \times 10^{-178} \) ]

→ i.e. most Hydrogen atoms will be in their ground state.

It is often appropriate to ignore all excited atomic states except the two states with energies \( E_1 \) and \( E_2 \)

→ can model the system as a collection of "2-level" atoms.

**Spontaneous Emission:**

An excited atom cannot remain in an excited state forever,

\[ \begin{array}{c}
\text{N}_i \\
\text{hv} \\
\text{N}_j
\end{array} \]

Can lose excess energy \((\text{by radiation})\) radiatively or non-radiatively. Radiatively \( \rightarrow \) photon of energy \( E = h\nu = E_2 - E_1 \) emitted in random direction, no phase relationship.

Transition rate = rate of change number of atoms in upper

\[ \text{state} = \frac{dN_2}{dt} = -A_{21}N_2 \]

Einstein A coefficient = prob. atom will make transition
Absorption:

\[ E_2 \rightarrow E_1 \]

Incident photon absorbed by atom in ground state.

Rate of transition rate (rate of change of # atoms in ground state) must be a function of:

1) Radiation field: \( W \)

2) \# atoms in lower level.

\[
\frac{dN_1}{dt}_{\text{abs}} = B_{12} N_1 W
\]

Stimulated Emission

\[ E_2 \rightarrow E_1 \]

Photons have same energy, direction, phase.

\[
\frac{dN_2}{dt}_{\text{st}} = -B_{21} N_2 W
\]

where \( B_{12}, B_{21} = \text{"Einstein B coefficient"} \).

Thermal Equilibrium (TE)

In TE, \( N_1 \) and \( N_2 \) do not change with time, so upward and downward transitions should occur at the same rate.

\[
\frac{dN_2}{dt}_{\text{st}} + \frac{dN_2}{dt}_{\text{sp}} = \frac{dN_1}{dt}_{\text{abs}}
\]

\( R_{\text{st}} \quad R_{\text{sp}} \quad R_{\text{abs}} \)
B_2 \text{N}_2 \text{U}_x = B_{21} \text{N}_2 \text{U}_x + A_{21} \text{N}_2$

Solving for \( \text{U}_x \) we have

\[
\text{U}_x = \frac{\text{N}_2 \text{A}_{21}}{\text{N}_1 \text{B}_{21} - \text{N}_2 \text{B}_{21}}
\]

Now in T.E., \( \text{N}_2 = \frac{\text{E}_2 - \text{E}_1}{\text{h} \nu / \text{e}} \cdot \text{e}^{-\text{h} \nu / \text{e}} \)

So 
\[
\text{U} = \frac{\text{A}_{21}}{\text{B}_{21}} \left( \frac{\text{B}_2}{\text{B}_{21}} \right) \left( \frac{\text{e}^{\text{h} \nu / \text{e}} - 1}{\text{e}^{\text{h} \nu / \text{e}} - 1} \right)
\]

But in T.E., \( \text{U} \) is given by BE function.

To agree with above, we must have \( \text{B}_{12} = \text{B}_{21} \)

[which means Prob. of stimulated emission \( (\propto \text{B}_{21} \text{U}_x) \) =
Prob. of absorption \( (\propto \text{B}_{12} \text{U}_x) \)]

and \( 2 \) \[
\text{A}_{21} = \frac{8\pi \text{h} \nu^3}{\text{c}^3} = \frac{8\pi \text{h} \nu}{\lambda^3}
\]

and \( 3 \) \[
\frac{\text{R}_{\text{st.}}}{\text{R}_{\text{sp}}} = \frac{\text{B}_{21} \text{N}_2 \text{U}}{\text{A}_{21} \text{N}_2} = \frac{1}{\left( \frac{\text{e}^{\text{h} \nu / \text{e}} - 1}{\text{e}^{\text{h} \nu / \text{e}} - 1} \right)}
\]

\[
\Rightarrow \text{Stimulated emission is negligible in T.E. at room temp}
\]

cut room temp \( \text{h} \nu / \text{e} \approx 2.5 \text{meV} \)

for optical light \( (\lambda = 0.6 \mu \text{m}) \), \( \text{h} \nu = 2 \text{eV} \)

and \( \text{h} \nu / \text{RT} = 80 \)

Even for a hot filament \( (T = 3000 \text{K}) \)

\[
\frac{\text{R}_{\text{st.}}}{\text{R}_{\text{sp}}} = \frac{1}{22,000} \Rightarrow \text{How do we make a laser?}
\]
Essential Elements of a Laser

1. Need to drive atoms

In T.E., if you put in incident light, (R) excites, depopulates lower state → \( \Delta N \) (can also excite upper state)

In T.E., more atoms in ground state → Stim obs. will occur more often than Stim emission

2. Need to drive atom out of T.E. and preferentially populate \( E_B \) level 2.

So \( N_2 > N_1 \) and \( R_{st} > R_{obs} \) → called population inversion

Essential elements:

1. Pump: external energy source that produces population inversion

2. Gain medium: gas (mixtures), liquid, solid

   (Lasers can be created out of half the known elements)

3. Optical cavity or resonator: “feedback” device

   \( \Delta > 10^9 \) reflectivity mirrors

Basic form: \( \text{gain medium} \)
A cavity will only support standing wave modes of wavelength \( \lambda \) that satisfy the condition:

\[
d = \frac{m \lambda}{2} \implies \gamma_m = \frac{m c}{2d}
\]

**Simplified Description of Laser operation:** (Generic description)

1. **Pump level**
2. **Non-radiative transition**
3. **Upper laser level** \( E_2 \)
4. **Light amplification**
5. **Upper laser level** \( E_2 \)
6. **Decay to ground level**

**4-Step Process:**

1. Energy from pump high enough to excite atoms from ground state to excited states, collectively labeled \( E_3 \).

2. Some atoms spontaneously decay all the way back to \( E_0 \).
   - But some many preferentially start the trip back by a very fast (usually radiativeless) transition to a "special" level \( E_2 \) the "upper laser level" or "metastable level" with lifetime \( T_{1/2} \approx 10^{-3} \text{s} \) (much longer than other levels).
   - Because it: long > atoms "pile up" at \( E_2 \) ("bottleneck") → \( N_2 \) gets bigger.

When level 2 does decay once by spontaneous emission, it does to level \( E_1 \) ("lower laser level"). \( E_1 \) is an ordinary level and quickly decay to \( E_0 \) → no pile-up of photons at level 1 → *population inversion* \( (N_2 > N_1) \).
Once population inversion is established, then when photon of resonant energy $E = E_2 - E_1 = h\nu$ passes by any of the $N_2$ atoms in upper laser level $\rightarrow$ Stim. emission can occur $\rightarrow$ amplification occurs. (Note: photon can also stimulate absorption from $1 \rightarrow 2$ but $N_2 > N_1$, so $R_{st.} > 1 \left( \frac{B_{21} N_2 \lambda_1}{B_{12} N_1 \lambda_2} \right)$ and occurs at each pass through gain medium.

Some atoms in $E_1 \rightarrow$ decay to $E_0 \rightarrow$ cycle continues. $\rightarrow$ steady population inversion.

Note: intensity increases through each pass through gain medium, but decreases each time it encounters output mirror $\rightarrow$ as long as gain $\frac{RT}{\text{loss per RT}}$ exceeds.

But as I gets bigger $\rightarrow N_2/N_1$ decreases $\rightarrow$ gain saturation.

Eventually as $I$ keeps increasing $\rightarrow$ gain per $RT = \text{loss per RT}$.

$N_2/N_1 = \text{const.} \rightarrow$ steady-state operation of laser.
Summary:

Laser process depends on:
1) Population inversion - through pump and requires existence of an upper level metastable state.
2) Seed photons \( \rightarrow \) from spontaneous emission which initiate stimulated emission.
3) Optical cavity \( \rightarrow \) amplification.
4) Saturation.
5) Output "coupler" mirror.

Characteristics of laser light:

1) Monochromaticity

Laser light much closer to monochromatic than any thermal light source. Stimulated emission produces identical \( \nu \) photons. But spontaneous emission has some lineshape function \( \nu(f) \) and some intrinsic width.

Output from laser: Stimulated + Some small spontaneous.

\( \rightarrow \) finite linewidth.

In practice \( \rightarrow \) much larger linewidth:

Changes in \( n \) due to mechanical vibrations, \( \rightarrow \) alter cavity length.

Remember: No resonance in cavity unless have nodes at mirrors.
\[ n = \frac{\# \text{ half wavelengths}}{2} \]

\[ \frac{n \lambda}{2} = L \quad \Rightarrow \quad n = \frac{2L}{\lambda} \]

\[ \text{If } \lambda = 500 \text{ nm, } L = 0.5 \text{ m}, \text{ then } n = 2 \times 10^6 \]

Suppose we went from \( n \) to \( n + 1 \)

By how much would \( \lambda \) have to change to "fit" in ?

\[ \lambda = \frac{2L}{n} \]

\[ \Delta \lambda = \frac{d\lambda}{dn} \Delta n = - \frac{2L}{n^2} \Delta n = - \frac{\Delta n \lambda}{n} \]

So for \( \Delta n = 1 \)

\[ \Delta \lambda = \frac{1}{2} \times 10^{-6} = 5 \times 10^{-7} \]

\[ \Delta \lambda = \frac{\lambda}{n} \frac{\lambda^2}{2L} \quad \Rightarrow \quad \Delta \lambda \text{ decreases as } \lambda \text{ increases.} \]

Also have doppler broadening.

\text{E.g. } \text{He-Ne: intrinsic } \Delta \nu = 1.56 \text{ Hz.} \]

But \( \text{by operational ranges are} 1 \text{ kHz to} 1\text{ MHz.} \)
1. Directionality:
   Laser output very directional because feedback only along cavity axis.
   → Spread due to diffraction.

   \[ w = \text{spot size}, \quad J = \lambda, \quad J = x/n \]

   Diffraction-limited divergence angle \( \theta_d \approx \frac{\lambda}{w} \)

   \( \theta_d = 0.15 \text{rad} \quad w = 1 \text{mm} \quad \theta_d = 0.03^\circ \)

2. Coherence:
   Can have \( L_B = 300 \text{m} \)

3. \[ l = \frac{\lambda}{n} \quad n = \frac{\lambda}{l} \]

   For \( n = 1 \), \( l = \lambda \quad \theta = \frac{\lambda}{L} \)

   Air: \( n = 1.0003 \quad \lambda = 0.63 \mu m \)

   \( v = 1.0 \text{cm}^{-1} \quad n = 1.0003 \quad \lambda = 0.63 \mu m \)

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